REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank)

2. REPORT DATE
5/28/05

3. REPORT TYPE AND DATES COVERED
Final Report for period 4/1/02-1/31/05

4. TITLE AND SUBTITLE
Swarming in two and three dimensions

5. FUNDING NUMBERS
DAAD19-02-1-0055

6. AUTHOR(S)
Andrea L. Bertozzi

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Duke University Department of Mathematics, Durham, NC 27708

8. PERFORMING ORGANIZATION
REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING / MONITORING
AGENCY REPORT NUMBER

43719. 12 -MA

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)
Final report for grant.

14. SUBJECT TERMS
UUV, surveillance, algorithms, cooperative control, image snake, boundary tracking, biological swarms, nonlinear PDE

15. NUMBER OF PAGES
5

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	**UNCLASSIFIED**	**UNCLASSIFIED**	**UL**

NSN 7540-01-280-5500

Standard Form 298 (Rev.2-89)
Prescribed by ANSI Std. 239-18
298-102

GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used for announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to ***stay within the lines*** to meet ***optical scanning requirements.***

Final Report for ARO grant DAAD19-02-1-0055

Andrea L. Bertozzi*
Department of Mathematics
Duke University
Durham, NC 27708

May 28, 2005

1 Statement of the Problem Studied

The future of military action will increasingly require new methods based on 'swarming' tactics in which a multitude of small units or 'pods' can operate in clusters with an overlaying network transmitting information.

This project aims at developing concise spatio-temporal models of the large-scale dynamics of swarm. The focus is on 'fluid-like' swarms in which the individual units have fairly distributed but localized density. The models have some connection to classical problems in fluid dynamics, with a potential for a richer structure arising from cooperativity between units in the swarm and from self-propulsion of individual units. The justification of the models is based on the internal dynamics swarming as opposed to classical principles of physical fluid flow. Models will be tested against numerical particle-based (Lagrangian) simulations and will be compared with known behavior from biological swarms such as locusts, ants, and fish. This biology-based portion of this research project will include collaboration with Mark Lewis, the Canada Research Chair of Mathematical Biology at the Univ. of Alberta.

The second part of this program involves bio-engineering motivated 'design of swarm'. We consider the inverse problem: given a large scale dynamics for a swarm, how can one design individual motion to achieve this outcome? Our approach is to start with continuum models designed to have desired solutions. We will use knowledge gained from the biological models to derive swarming algorithms that could have both military and industrial use. The designed swarms will include an additional component not present in biological models, that of a communications network distributed among the swarmer subgroups that will facilitate operations.

*Current address: Department of Mathematics, University of California Los Angeles, Los Angeles, CA 90095

2 Summary of the most important results

2.1 Biological swarm models

We construct a continuum model for the motion of biological organisms experiencing social interactions and study its pattern-forming behavior. The model takes the form of a conservation law in two spatial dimensions. The social interactions are modeled in the velocity term, which is nonlocal in the population density and includes a parameter that controls the interaction length scale. The dynamics of the resulting partial integrodifferential equation may be uniquely decomposed into incompressible motion and potential motion. For the purely incompressible case, the model resembles one for fluid dynamical vortex patches. There exist solutions which have constant population density and compact support for all time. Numerical simulations produce rotating structures which have circular cores and spiral arms and are reminiscent of naturally observed phenomena such as ant mills. The sign of the social interaction term determines the direction of the rotation, and the interaction length scale affects the degree of spiral formation. For the purely potential case, the model resembles a nonlocal (forwards or backwards) porous media equation. The sign of the social interaction term controls whether the population aggregates or disperses, and the interaction length scale controls the balance between transport and smoothing of the density profile. For the aggregative case, the population clumps into regions of high and low density. The characteristic length scale of the density pattern is predicted and confirmed by numerical simulations.

We consider the potential case above in which short range repulsion is modelled locally and depends nonlinearly on the local density. For the case o f one spatial dimension, we study the steady states analytically and numerically . There exist strongly nonlinear states with compact support and steep edges tha t correspond to localized biological aggregations, or clumps. In the limit of la rge population size, the clumps approach a constant density swarm with abrupt ed ges. We use energy arguments to understand the nonlinear selection of clump solu tions, and to predict the internal density in the large population limit. Numeri cal simulations reveal dynamic coarsening behavior, in which small clumps form r apidly, and then merge over longer time scales. Simulations for the case of two spatial dimensions also reveal clumping behavior.

2.2 The design of swarm behavior

2.2.1 Boundary tracking

We develop a model for the self-organizing, decentralized, real-time motion planning for a swarm of homogeneous mobile robots in a stationary environment. The model allows the robots to coope ratively locate the boundary of a given environmental function in two space dimensions using a combi nation of sensing and communication. Starting from a partial differential equation (PDE) used in image processing for edge detection, a finite difference approximation provides the movement rules for each robot. Each node in the discretization corresponds to a robot in the environment. We consider physical parameters for a specific platform of underwater vehicles. We design the algorithm to function with asynchronous communication and noisy position information. We present numerical simulations illustrating the stability and performance of this system.

We modify this method to make it practical for testbed implementation with only binary sensors. Such general problems are of current interest for unmanned vehicle operations with specific applications ranging from coastal algae blooms, chemical plumes, and adaptive ocean sampling to future applications including oil spills or hazardous chemicals. We implement this algorithm on the CalTech Multi-Vehicle Wireless Testbed.

2.2.2 Searching

We consider both greedy-auction (deterministic) and stochastic (Levy) searching strategies for cooperative UAVs. The greedy method is shown in simulation to have good scaling properties with the system size and number of targets. For the Levy problem we show that when searching for targets with a priori information, biasing the search direction as opposed to the path length provides the most efficient search strategy. The greedy algorithm is implemented on two different platforms on the CalTech Multi-Vehicle Wireless Testbed.

2.2.3 Virtual potentials

We consider a motion planning method based on cooperative biological swarming models with virtual attractive and repulsive potentials (VARP). We derive a map between the model and fan speeds for the Kelly, a second order vehicle on the Caltech Multi Vehicle Wireless Testbed. The motion planning map results leads to the development and implementation of a point to point controller which is subsequently used as part of a cooperative searching algorithm. The VARP control method is scalable and can be used to organize a swarm of robotic vehicles.

3 Publications, Preprints, and Technical Reports

3.1 Papers published in peer reviewed journals

1. Chad Topaz and Andrea L. Bertozzi, Swarming Patterns in a Two-Dimensional Kinematic Model for Biological Groups, *SIAM Journal on Applied Mathematics* Volume 65, Number 1 pp. 152-174, 2004.

3.2 Papers published in conference proceedings or non-peer reviewed journals

1. M. Kemp, A.L. Bertozzi, D. Marthaler, Multi-UUV perimeter surveillance, edited by C., Brancart, E. An, M. Benjamin, 2004 IEEE/OES Workshop

on Autonomous Underwater Vehicles, Sebasco Estates, Maine, pages 102-107.

2. A.L. Bertozzi M. Kemp, and D. Marthaler, Determining Environmental Boundaries: Asynchronous communication and physical scales, Cooperative Control, Lecture Notes in Control and Information Systems, V. Kumar, N. Leonard, and A. S. Morse eds, vol. 309, pages 25-42, 2004.

3. Daniel Marthaler and Andrea L. Bertozzi, Tracking environmental level sets with autonomous vehicles, S. Butenko, R. Murphey and P.M. Pardalos (editors), "Recent Developments in Cooperative Control and Optimization", Kluwer Academic Publishers, 2003 [htm].

4. B. Cook, D. Marthaler, C. Topaz, A. Bertozzi, and M. Kemp, Fractional bandwidth reacquisition algorithms for VSW-MCM, Multi-Robot Systems: From Swarms to Intelligent Automata, Volume II, 2003, Kluwer Academic Publishers, Dordrecht, A.C. Schultz et al. eds. 77-86 .

3.3 Papers presented at meetings, but not published in conference proceedings

None.

3.4 Manuscripts submitted, but not published

1. Chung H. Hsieh, Zhipu Jin, Daniel Marthaler, Bao Q. Nguyen, David J. Tung, Andrea L. Bertozzi, and Richard M. Murray, Experimental Validation of an Algorithm for Cooperative Boundary Tracking , to appear in the American Control Conference, 2005.

2. B. Q. Nguyen, Y-L Chuang, D. Tung, C. Hsieh, Z. Jin, L. Shi, D. Marthaler, A. L. Bertozzi, R. M. Murray, Virtual attractive-repulsive potentials for cooperative control of second order dynamic vehicles on the Caltech MVWT, to appear in the American Control Conference 2005.

3. Chuang, Y.L., Oren, R., Bertozzi, A.L., Phillips, N., and Katul, G.G., "The porous media model for the hydraulic system of a tree: from sap flux data to transpiration rate", to appear in Ecological modeling.

4. C.M. Topaz, A.L. Bertozzi, and M.A. Lewis. A nonlocal continuum model for biological aggregation. Submitted to the Bulletin of Mathematical Biology, 2005.

5. D. Marthaler, A. Bertozzi, and I. Schwartz, Levy searches based on a priori information: The Biased Levy Walk, submitted to Physical Review E, 2004.

3.5 Technical reports submitted to ARO

1. Daniel Marthaler and Andrea L. Bertozzi, Collective motion algorithms for determining environmental boundaries, 2002.

4 List of all participating scientific personnel showing any advanced degrees earned by them while employed in the project

1. Andrea Bertozzi, Professor of Mathematics, UCLA, and Professor of Mathematics and Physics, Duke University (Principal Investigator)
2. Mark Lewis, Canada Research Chair of Mathematical Biology, Univ. of Alberta
3. Mathieu Kemp, Director of Physics, Nekton Research LLC, Durham, NC
4. Richard Murray, Professor, California Institute of Technology
5. Maria D'Orsogna, Assistant Researcher, UCLA 2004-present
6. Daniel Marthaler, Assistant Researcher, UCLA, 2003-4, Postdoc, Duke University, 2002-3
7. Chad Topaz, VIGRE Assistant Professor, Duke Univ. and UCLA, 2002-present
8. Dejan Slepcev, Assistant Researcher and Adjunct Assistant Professor, UCLA, 2004-present
9. Yao-Li Chuang, MS in Physics, Duke University (also current PhD student)
10. Zhipu Jin, PhD student in Control and Dynamical Systems, CalTech
11. Ling Shi, PhD student in Control and Dynamical Systems, CalTech
12. Benjamin Cook, BS in Mathematics and Physics, Duke University 2003
13. Chung Hsieh, BSE in Electrical Engineering, UCLA 2004
14. Bao Nguyen, BSE in Electrical Engineering, UCLA 2005
15. David Tung, BSE student in Electrical Engineering, UCLA 2004